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Using a parametric approach to understand changes in risk

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Contents

Introduction

Risk results can change for many reasons: trading activity that generates exposure to new factors, changes in exposures to existing factors, changes in risk factor volatilities, or changes in correlation among risk factors. Understanding changes in risk estimates can be key, especially in times of crisis when volatilities spike and correlations point in the same direction, eliminating the diversification that was supposed to protect a portfolio. Understanding those changes also enables us to determine whether portfolio management over the period has increased or reduced risk. Yet understanding those changes, especially for simulation-based analytics, can seem impossible.

The parametric method makes understanding these changes relatively simple. This method is valid for a large range of portfolios exhibiting small or negligible convexity. The parametric approach and the closed formula used to derive portfolio volatility lend themselves to easy mathematical calculations that can typically provide the sensitivity of the risk function to the parameters being used: exposures, factor volatilities, and correlations. The methodology proposed here decomposes the variation in risk through a simple Taylor expansion of the parametric-risk function. Using first and second order sensitivities, the methodology works in a similar fashion for performance attribution or P&L explanation. When the residual part is small, the method is effective and appealing because effects computed are additive.

This paper first describes the methodology and then details the types of outputs and analysis that can be generated using Axioma Risk™: our enterprise portfolio risk management system. We extracted the data from Axioma Risk with our REST API and generated the charts with Python libraries.

Methodology

We will explain a variation in risk through a simple first and second order expansion of the volatility parametric function. Risk is defined as in (1).

$$v = \sqrt{\sum_{k,l} d_k d_l\, \rho_{kl} \sigma_k \sigma_l + \sum_{i \in I} e_i^2 s_i^2 + \sum_{i,j \in J} e_i e_j r_{ij} s_i s_j} \tag{1}$$

The vector $(d_i)_{i=1IN}$ represents the sensitivities to risk factors. When the covariance matrix is decomposed as $\Sigma = VCV$, $V = (\sigma_i)_{i=1\ldots N}$ represents the factor volatilities and $C = \left(\rho_{ij}\right)_{i,j}$ the correlation matrix. N is the number of systematic factors and M the number of specific risk sources. Specific risk components are generated either by independent sources I and correlated ones J: we have the specific exposures $(s_i)_{i \in I,J}$, the specific volatilities $(e_i)_{i \in I,J}$, and the correlation between specific risk factors $\left(r_{ij}\right)_{i,j \in J}$.

We are interested in changes in v between two dates, t_1 and t_2. We denote $\bar{v}(t_2)$ as the risk of the portfolio evaluated at t_2 but by taking into account only the factors that were also present on t_1. The variation in risk can be written as:

$$v(t_2) - v(t_1) = \left(v(t_2) - \bar{v}(t_2)\right) + \left(\bar{v}(t_2) - v(t_1)\right). \tag{2}$$

The first part can be explained by newly traded positions (and factors), and we focus on the second part $\left(\bar{v}(t_2) - v(t_1)\right)$ to explain the variation at first order (and second order for some) thanks to partial derivatives to exposures, volatilities, and correlations.

The effects we will compute between the dates t_1 and $t_2$will then be computed as:

Exposure effect:

> First order:

> $exp = \sum_{i=1}^{N} \frac{\partial v}{\partial d_i} \left(d_i(t_2) - d_i(t_1) \right)$

> Second order:

> $exp_2 = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial^2 v}{\partial d_i \, \partial d_j} \left(d_i(t_2) - d_i(t_1) \right) \left(d_j(t_2) - d_j(t_1) \right)$

Volatility effect:

> First order

> $vol = \sum_{i=1}^{N} \frac{\partial v}{\partial \sigma_i} \left(\sigma_i(t_2) - \sigma_i(t_1) \right)$

> Second order:

> $vol_2 = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial^2 v}{\partial \sigma_i \, \partial \sigma_j} \left(\sigma_i(t_2) - \sigma_i(t_1) \right) \left(\sigma_j(t_2) - \sigma_j(t_1) \right)$

Correlation effect:

> $cor = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial v}{\partial \rho_{ij}} \left(\rho_{ij}(t_2) - \rho_{ij}(t_1) \right)$

Specific exposure effect:

> First order:

> $spec\_exp = \sum_{i=1}^{M} \frac{\partial v}{\partial s_i} \left(s_i(t_2) - s_i(t_1) \right)$

> Second order:

> $spec\_exp_2 = \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} \frac{\partial^2 v}{\partial s_i \partial s_j} \left(s_i(t_2) - s_i(t_1) \right) \left(s_j(t_2) - s_j(t_1) \right)$

Specific volatility effect:

> $spec\_vol = \sum_{i=1}^{M} \frac{\partial v}{\partial e_i} \left(e_i(t_2) - e_i(t_1) \right)$

The derivatives are indicated in the Appendix.

The information is extracted through a correlation statistic from Axioma Risk that contains all those parameters on any given date: exposures, factor volatilities, and factor correlations.

Explaining changes in risk

This methodology has the big advantage of explaining changes linearly by effect down to the factor and through time.

Changes over a given single period

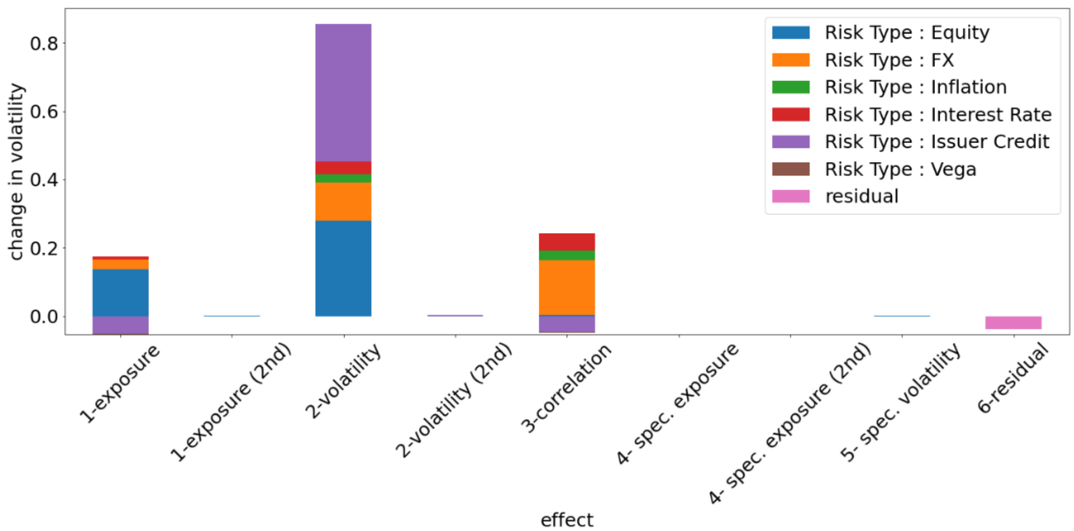
We can decompose the changes in risk between two dates and get insights into what drove those changes. We have included some simple charts to show:

> What changed by risk type and by effect type (exposure, volatility, correlation etc.)

> The breakdown of a given effect by risk type or factor type

For example, below is the change in risk for a multi-asset portfolio between March 18 and March 25, when volatility rose from 9.68% to 10.83%. The 1.2% change can then be decomposed as illustrated in Figure 1. Each effect type can be further decomposed by risk type. The residual part is the part in the change that remains unexplained:

Figure 1. Change in risk explained by effect type



Source: Axioma Risk

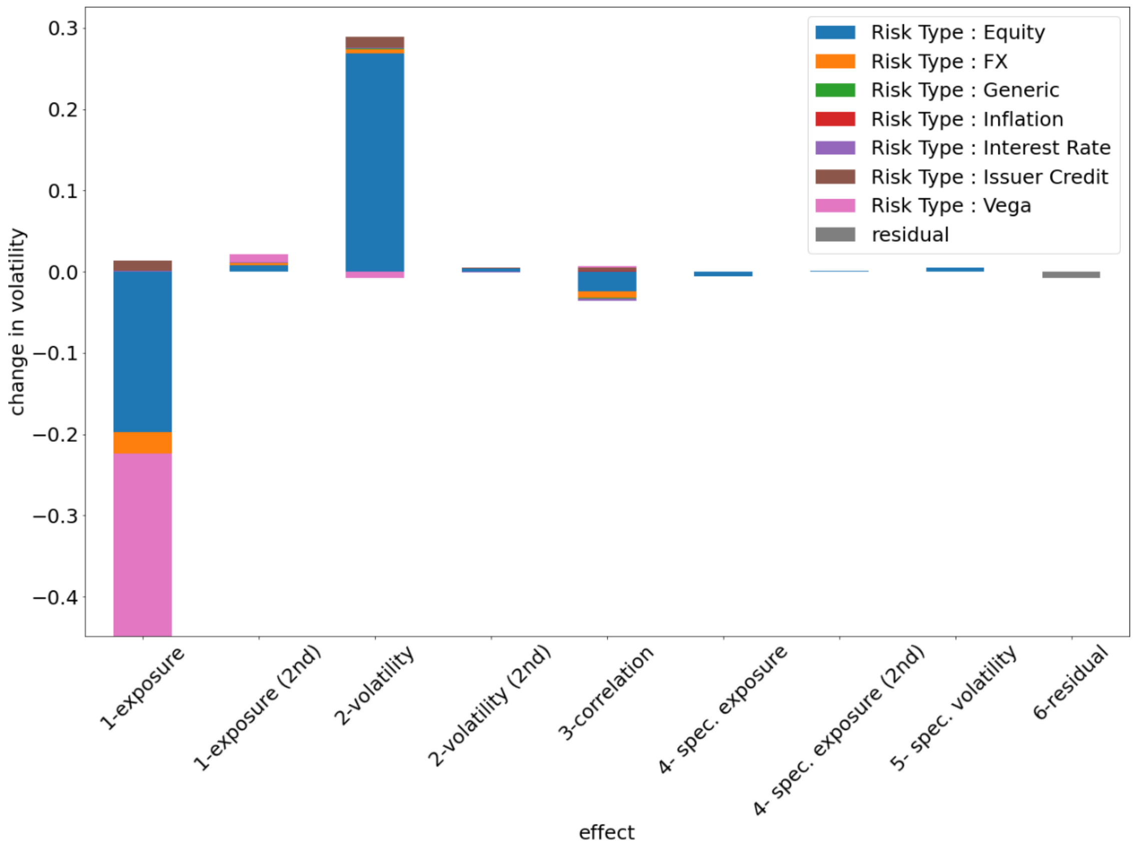
The exposure effect

The exposure effect captures the impact of changes in exposures between two dates. Such changes can be due to two sources: trading resulting in exposure changes, or exposures changing naturally (when equity loadings change, sensitivities such as equity delta or durations change through time, etc.). We would expect the biggest component to come from the former. Trading activity impacts exposures to existing factors and can also occasionally introduce exposure to new factors. The factor set is relatively constant for a given portfolio across time in Axioma Risk (equity fundamental factors, interest rate, and fixed-income factors), and the new-factor effect tends to be marginal in practice. This effect would be captured by the first term of (2).

We have introduced a second order effect to capture the exposure effect more accurately. This yields a significantly better explanation (i.e., reducing the residual) in some cases where trading abruptly changes exposures.

Therefore, the exposure effect measures the portfolio-management impact to the risk of the portfolio and can answer the question: Has trading led to more risk or lowered it by reducing positions or introducing hedges?

Figure 2. Exposure effect showing hedges offsetting increase in volatility effect

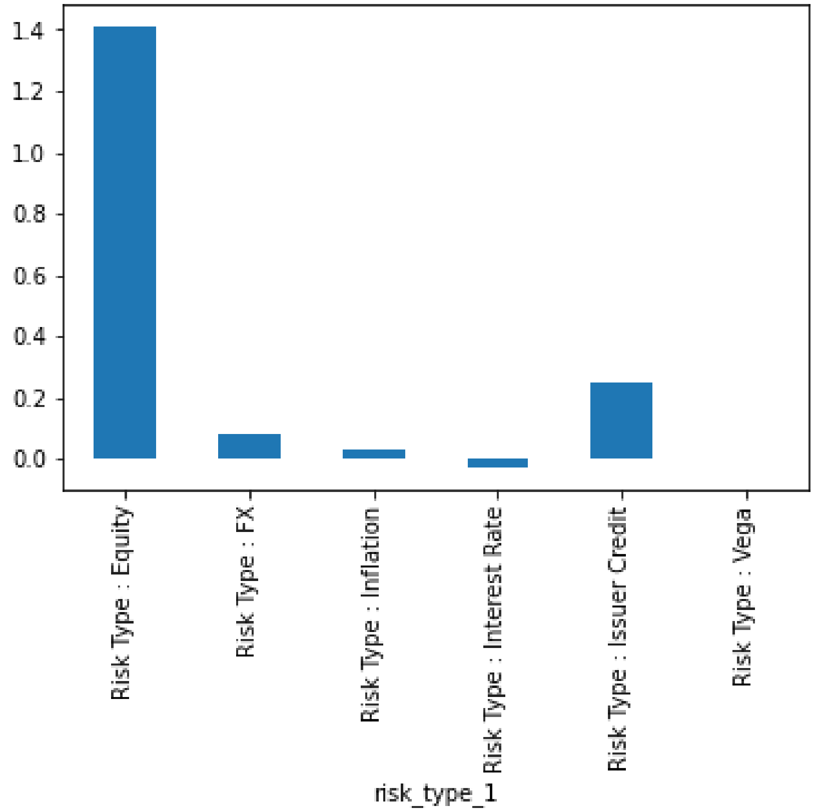


Source: Axioma Risk

The volatility effect

The volatility effect naturally captures changes in factor volatilities, which can be crucial in times of crisis when volatility shoots up, in particular for equity and credit spreads. While volatilities can change significantly, those changes are in general gentle compared with exposure changes. As a result, while we have introduced a second order effect, it is generally muted (we have tested standard Axioma Risk settings that would correspond to a medium horizon). A more aggressive weighting scheme could make the second order effect become more significant. The chart below gives a further decomposition of the volatility effect seen in Figure 1:

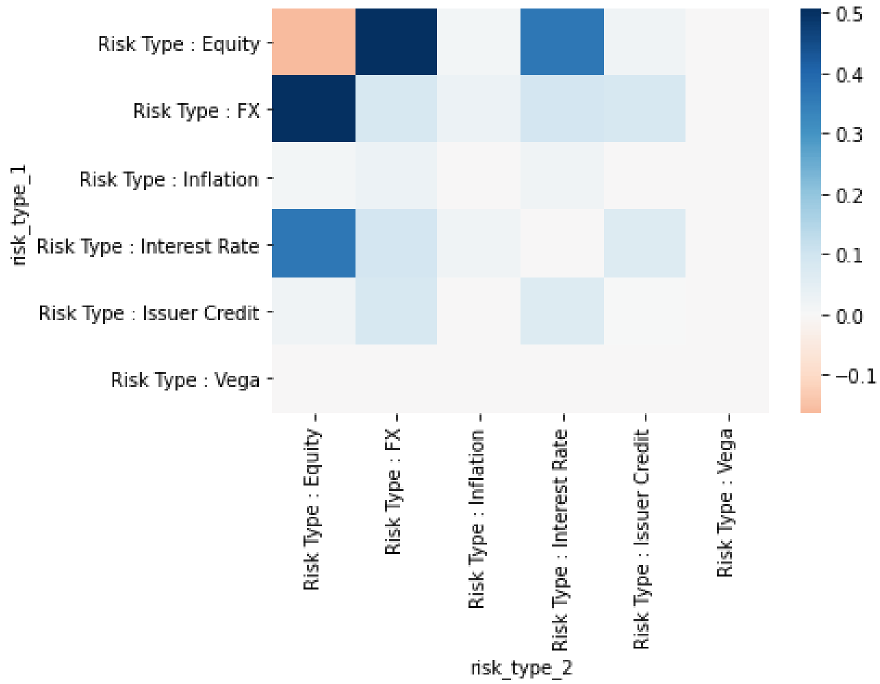
Figure 3. Volatility effect by risk type on a multi-asset class portfolio



The correlation effect

Although the correlation effect is usually not as significant as the previous two, it should be monitored because it indicates how changes in factor correlation impact portfolio risk. In normal circumstances, low or moderate correlation between factors yields a diversification effect that protects the portfolio. In times of crisis, correlation changes dramatically and tends toward 1 in absolute terms, losing this natural protection. The correlation effect captures these changes. It is a two-dimensional effect, and a heat map visualizes which correlation moves impacted the overall risk number. In the example on the next page, correlation between equity factors and currency factors decreased risk while correlation among equity factors increased risk.

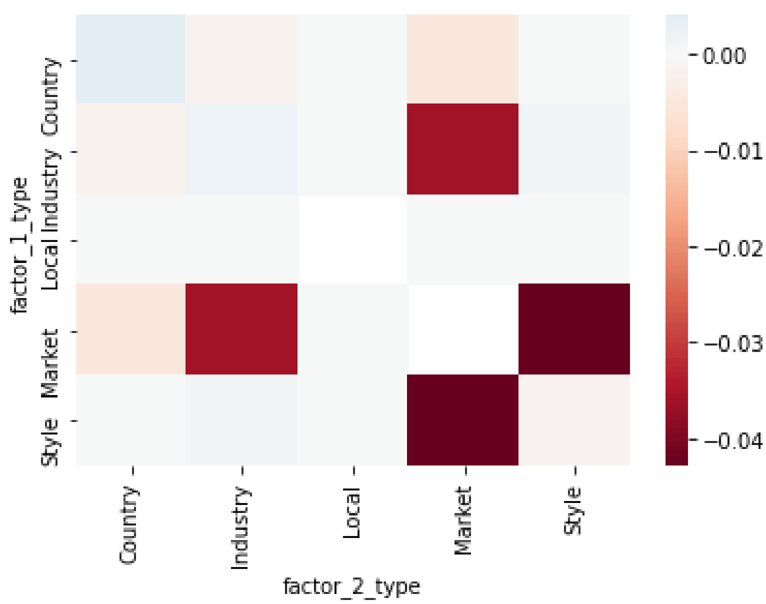
Figure 4. Breakdown of correlation effect by risk type



Source: Axioma Risk

We can then drill down further. For example, the negative correlation effect within the equity factors can be explained further across country, industry, style, and market factors:

Figure 5. Further breakdown: Which correlations among equity factors had an impact on risk?



Source: Axioma Risk

Cumulative effects over multiple periods

The methodology produces effects that are additive and lends itself naturally to cumulating those effects through time. T is the number of periods, which could be weekly, for example.

$$cumulative\ exposure\ effect = \sum_{t=0}^{T-1} exp(t,t+1) + exp_2(t,t+1)$$

$$cumulative\ volatility\ effect = \sum_{t=0}^{T-1} vol(t,t+1) + vol_2(t,t+1)$$

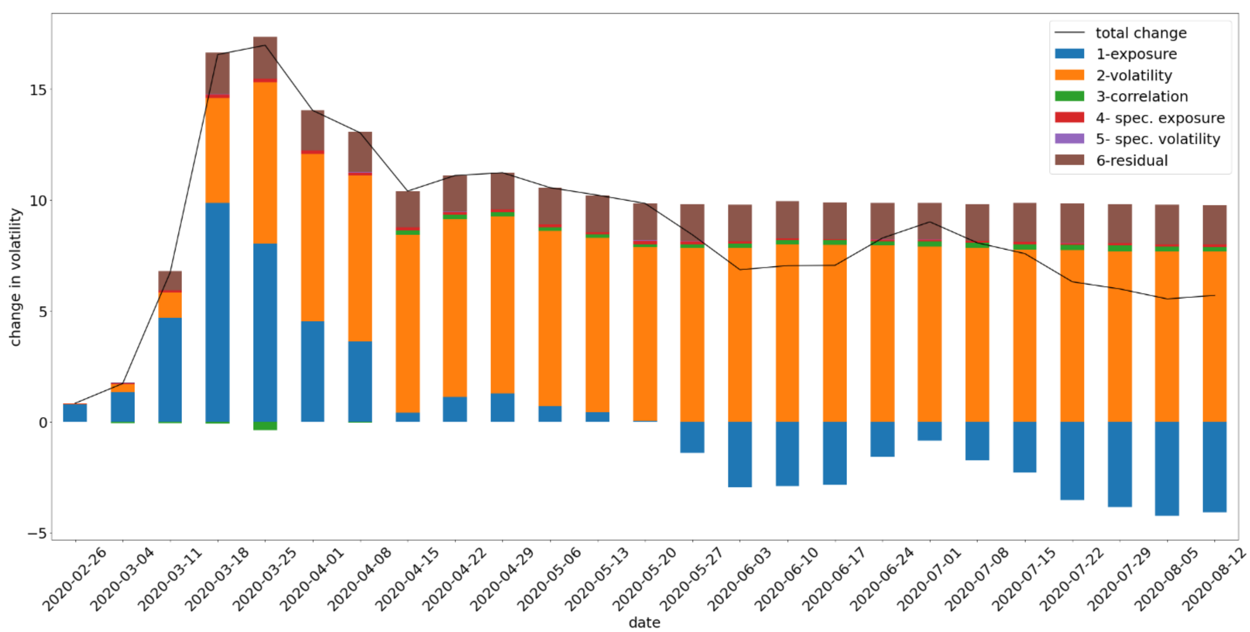
$$cumulative\ correlation\ effect = \sum_{t=0}^{T-1} cor(t,t+1)$$

$$cumulative\ spec\ exposure\ effect = \sum_{t=0}^{T-1} spec\_exp(t,t+1)\ +\ spec\_exp_2(t,t+1)$$

$$cumulative\ spec\ volatility\ effect = \sum_{t=0}^{T-1} spec\_vol(t,t+1)$$

Below is an example of a HY portfolio using the new granular issuer curves from Axioma Risk. DTS is the credit sensitivity. As spread spikes, so do the DTS exposures and so does the risk estimate:

Figure 6. The DTS effect can be observed clearly through the cumulative exposure effect



Source: Axioma Risk

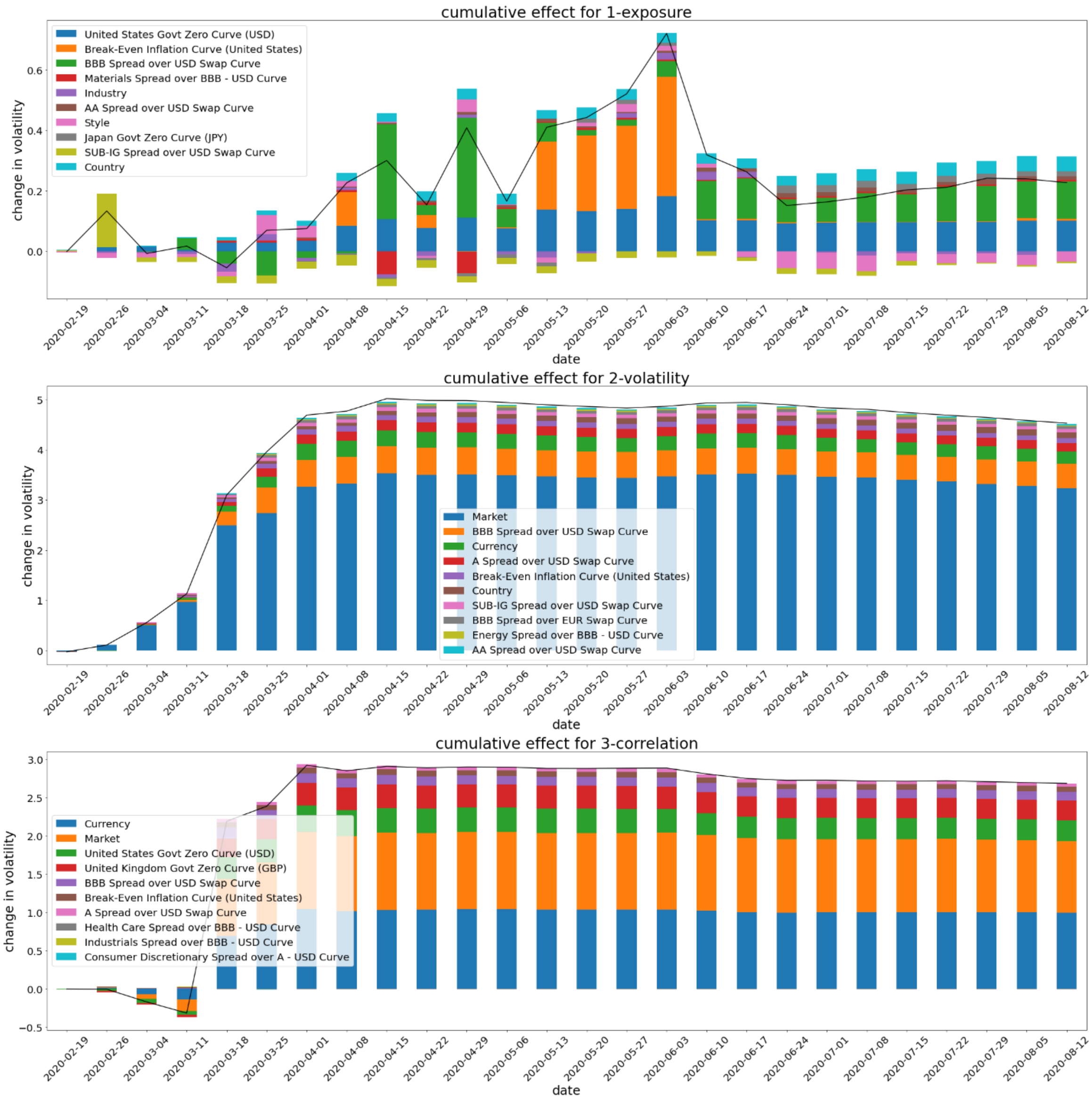
For a multi-asset portfolio, we can also visualize the cumulative effect broken down by risk type, factor type, or even factor. When there are too many factor types to display, we can choose only the most meaningful by finding iteratively a group A of F factors or factor types where the contribution of those factors is as close as possible to the total cumulative effect.

If $e(t) = e(t, t+1) = \sum_{i=1}^{F} e_i(t)$ is the total effect over the F factors/factor types considered between the time t and t+1, we can call $e_A(t) = \sum_{i \in A} e_i(t)$ the effect by counting only the selected factors. We then find iteratively and in a greedy fashion the new factor or factor type that minimizes over the whole period the sum of square differences:

$$\sum_{t=0}^{T-1} (e(t) - e_A(t))^2$$

Figure 7 shows the cumulative effects for exposure, volatility, and correlation through time when we count only the 10 most important factor types. The black line represents the total effect.

Figure 7: Cumulative effects explained by factor type: Taking the 10 most meaningful of them is sufficient to understand what happened during that period



Source: Axioma Risk

Conclusion

When risk is an important constraint in the portfolio management process, understanding why it increases in times of heightened volatility is key. If we can decompose a change in risk into the three main effects (exposure, volatility, and correlation), we can easily see if exposures to risk factors have been managed to reduce risk, which factor volatilities are driving the volatility effect, and which correlations have removed the diversification effect. Our approach to changes in risk estimates is based on a Taylor expansion of the parametric risk function and captures those exposure, volatility, and correlation effects. These effects are additive down to the factor level and can yield insightful analysis of a given portfolio through time. The only limitation is that the risk estimate must use the parametric approach, which is valid only for low-convexity portfolios. However, most portfolios in the asset management industry fall into this category.

Appendix A: Derivatives Formulas

$$v = \sqrt{\sum_{k,l} d_k d_l \rho_{kl} \sigma_k \sigma_l + \sum_{i \in I} e_i^2 s_i^2 + \sum_{i,j \in J} e_i e_j r_{ij} s_i s_j}$$

It is useful to define the intermediate quantities $f_i = \sum_{l=1}^{N} d_l \rho_{il} \sigma_l$ and $g_i = \sum_{l \in J} s_l r_{il} e_l$. The formulas for all the derivatives of the risk function are given below:

$$\frac{\partial v}{\partial d_i} = \frac{\sigma_i f_i}{v} \, (3.1)$$

$$\frac{\partial v}{\partial \sigma_i} = \frac{d_i f_i}{v} \, (3.2)$$

$$\frac{\partial v}{\partial \rho_{ij}} = \frac{d_i d_j \sigma_i \sigma_j}{2v} \, (3.3)$$

The sensitivities to specific exposure and volatility are as follows respectively (I the set of simple specific risk factors and J the set of correlated specific risk factors):

$$\frac{\partial v}{\partial s_i} = \frac{s_i e_i^2}{v}, i \in I \, (3.4.1)$$

$$\frac{\partial v}{\partial s_i} = \frac{e_i g_i}{v}, i \in J \, (3.4.2)$$

$$\frac{\partial v}{\partial e_i} = \frac{e_i s_i^2}{v}, i \in I \, (3.5.1)$$

$$\frac{\partial v}{\partial e_i} = \frac{s_i g_i}{v}, i \in J \, (3.5.2)$$

In most cases, first order effects are sufficient to explain reasonably well the change in risk. In other cases of a portfolio actively managed with significant changes in exposures, a second order term is needed to correct the first order effect and yield a better explanation. The second order sensitivity to exposures can be written as:

$$\frac{\partial^2 v}{\partial d_i \, \partial d_j} = \frac{\sigma_i \sigma_j}{v} \left(\rho_{ij} - \frac{f_i f_j}{v^2} \right) (3.6.1)$$

The second order sensitivity to volatilities can be written similarly:

$$\frac{\partial^2 v}{\partial \sigma_i \, \partial \sigma_j} = \frac{d_i d_j}{v} \left(\rho_{ij} - \frac{f_i f_j}{v^2} \right) (3.6.2)$$

And the second order effect to specific exposures can be written as ($\delta_{ij} = 1 \; if \; i = j, 0 \; otherwise$):

$$\frac{\partial^2 v}{\partial s_i \, \partial s_j} = \frac{1}{v} \left(\delta_{ij} e_j^2 - \frac{s_i s_j e_i^2 e_j^2}{v^2} \right), i, j \in I (3.7.1)$$

$$\frac{\partial^2 v}{\partial s_i \, \partial s_j} = \frac{e_i e_j}{v} \left(r_{ij} - \frac{g_i g_j}{v^2} \right), i, j \in J (3.7.2)$$

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Learn more about how Qontigo can help you better manage risk and enhance your investment process.
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